



Using numeracy across curriculum help booklet

INTRODUCTION

Numeracy in your lessons should not be an 'add-on' but should enhance what you are doing already.

Many numeracy skills are being taught implicitly in lessons, so you can easily raise the profile by using the numeracy logo.



Simple things like reducing the amount you allow pupils to use a calculator and getting them to use paper methods especially for whole number calculations will make a difference.

Use estimation in your lessons and highlight to pupils that some numbers are rounded, e.g. the number of people at a football match is rounded to nearest 1000.

INCLUDED IN THIS BOOKLET IS A COMPILATION OF METHODS USED IN MATHS CLASSROOMS AND SOME USEFUL INFORMATION ABOUT THE LEVELS PUPILS ARE WORKING AT IN MATHS IN YOUR LESSONS.

NUMBER - CALCULATIONS

You can promote numeracy in your lessons by encouraging pupils to calculate without a calculator especially with whole numbers.

or

8642

+753

9395

1

pupils to use at St. Anne's.

By this time they

understand

carrying

The third method is what we expect the majority of

Addition

Example: 8642 + 7538000 + 600 + 40 + 2700 + 50 + 38000 + 1300 + 90 + 5

or

8000 + (600 + 700) + (40 + 50) + (2 + 3) = 8000 + 1300 + 90 + 5 = **9395**

= 9395

Subtraction

Example:	2410 - 482	or		
·	$ \begin{array}{r} 1000 \\ 2000 + 400 + 10 + 0 \\ \underline{400 + 80 + 2} \\ 1000 + 900 + 20 + 8 \\ = 1928 \end{array} $		1 13 10 1 2410 <u>- 482</u> 1928	By now they understand <i>borrowing</i>

The second method is what we expect the majority of pupils to use at St. Anne's. Some pupils may need to use a number line and add up but very few.

Multiplication

Grid multiplication:

Example: 24 x 16

	х	20	4	
	10	200	40	
	6	120	24	
26 x 16	5 = 200 +	120 + 4	10 + 24	= 384

Formal multiplication methods:

24	×	6	becomes
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		2	4
	×		6
	1	4	4
		2	
A	nsw	er:	144

This is the method which is often currently taught in primary school; however the new national curriculum states that pupils must use formal methods, which does not include the grid method.

$24 \times$	16	becomes
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	2 2	4
×	1	6
2	4	0
1	4	4
3	8	4

Answer: 384

Division

Chunking:

Example: 432 ÷ 15

<u>20 + 8</u>	
15)432	15 x 10 = 150
(15 x 20) 300	15 x 20 = 300
132	$15 \times 4 = 60$
(15 x 8) 120	15 x 8 = 120

∴ 432 ÷ 15 = 28 remainder 12

The above method is referred to as '*chunking'*, as they are subtracting chunks of 15 at a time.

Formal methods:

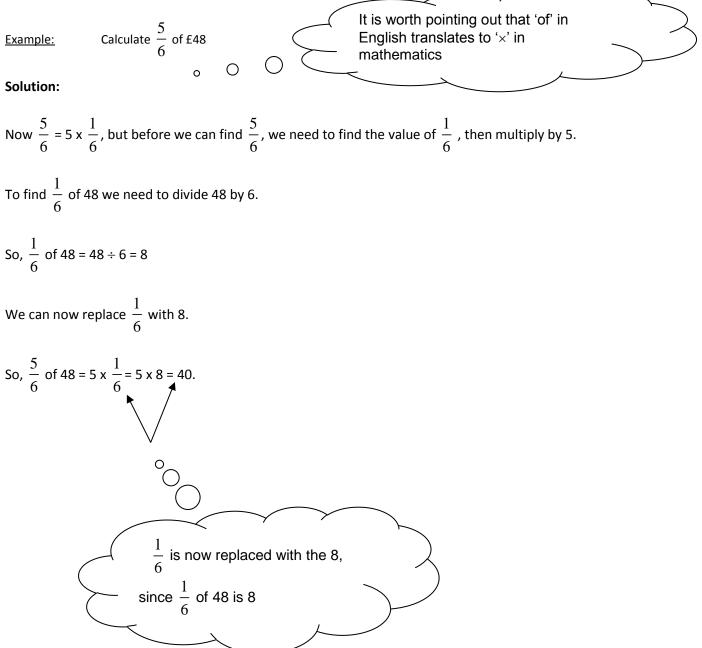
Short division 98 ÷ 7 becomes 432 ÷ 5 becomes 496 ÷ 11 becomes 1 4 6 r 2 4 5 r 1 8 3 1 1 4 2 9 6 7 9 8 5 3 Answer: $45\frac{1}{11}$ Answer: 14 Answer: 86 remainder 2

Long division

432 ÷ 15 becomes 432 ÷ 15 becomes 432 ÷ 15 becomes 2 8 r 12 2 8·8 2 8 1 5 4 3 2 . 0 2 3 2 1 5 4 3 1 5 4 3 15×20 3 0 0 0 0 3 0 ↓ 1 3 2 1 3 2 1 3 2 15×8 0 0 1 20 1 2 1 2 1 2 2 2 0 1 1 12 0 0 $\frac{12}{15} = \frac{4}{5}$ Answer: $28\frac{4}{5}$ Answer: 28 remainder 12 Answer: 28-8

This is the method which is often currently taught in primary school; however the new national curriculum states that pupils must use formal methods, which does not include the 'chunking' method.

Calculating with fractions



Final answer

$$\frac{5}{6}$$
 of £48 = £40

Calculating with Percentages

Example: Calculate the VAT on £240 @ 17.5%

Solution:

Noting first that 17.5% = 10% + 5% + 2.5%

Now 10% of £240 = $240 \div 10 = £24$ •

Now, since 10% is equal to £24

and ½ of 10% is 5%

then

5% of £24

=½ of £24

=£12

Similarly,

2.5% of £24 = £6.

We have then: 17.5% of £240 = [10% + 5% + 2.5%] of £240

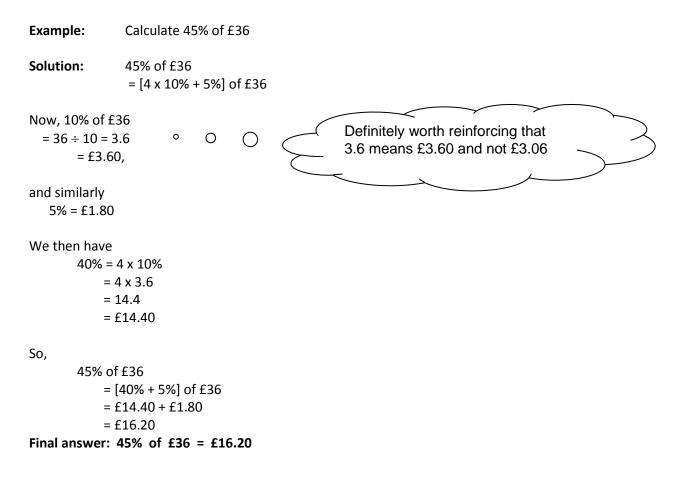
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Since $10\% = \frac{1}{10}$, to find 10% of £240 we divide 240 by 10.

= £42

We can of course use this method to calculate any percentage but this method is best used when the numbers are "nice". See the example below.



Whilst it is possible, it would be inefficient to calculate 23.608% of 406.87kg using this method. For this a calculator is best used!

Calculator Methods

Calculator methods generally involve changing the fraction or percentages into a decimal first. Decimals are often referred to as "multiplying factors" since, having turned the fraction or percentage into a decimal, we use it to multiply.

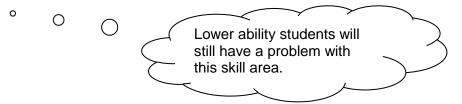
Changing a fraction into a decimal

To change a fraction into a decimal, you 'divide the top number (numerator) by the bottom number (denominator)'. To help pupils to remember this it is often useful to remind them that a fraction looks like a divide sign:



Changing a percentage into a decimal

To change a percentage into a decimal we aim to remember that percentage means "per-hundred". Therefore 23% means 23 per 100. This is written as $^{23}/_{100}$, and to change this fraction into a decimal we divide 23 by 100, as above.



Note:

Most pupils simply need to remember that to change a percentage into a decimal, they should divide the percentage by 100.

A "starter" to a lesson where this skill needs to be utilised might look like this, where the pupils need to fill in the blank spaces with the appropriate value.

	As this is a recurring decimal, how	
(many decimal places do we need to	
	write down? It is worth pointing out	
	that you would use all the numbers on	
	the calculator and round off at the end	
\sim	of the calculation only.	-
	\mathcal{F}	-
\searrow		

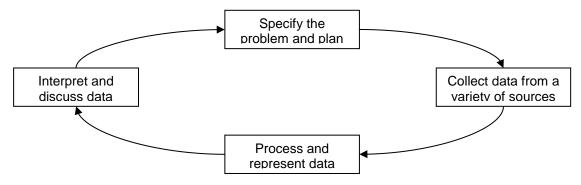
Fraction	Decimal	Percentage
4/5	?	?
?	0.125	?
?	?	65%
8/13	?	?

Example:	Calculate 5.35% of 23,456kg
Solution:	Find 1% 23456 ÷ 100 = 234.56 ° ° ° This is the
So, 234.56	5 x 5.35 = 1,254.896
Final answer 5.35% of 2	23,456kg = 1,254.896kg

Depending upon the context, a suitable degree of accuracy would now be required.

HANDLING DATA - INTRODUCTION

Throughout the mathematics curriculum the clear message is that data handling is best taught in the context of real statistical enquiries, in a coherent way so that teaching objectives arise naturally from the whole cycle, as represented in the following diagram:



Other subjects within the national curriculum have similar descriptions for the role of data handling within their subject-specific contexts:

Science: Test explanations by using them to make predictions and by seeing if evidence matches the predictions.
Use first-hand and secondary data to carry out a range of scientific investigations, including complete investigations;
Information and Communications Technology (ICT): Pupils should be taught how to collect, enter, analyse and evaluate information relevant to the enquiry and reach conclusions;

History: Identify, select and use a range of appropriate sources of information, evaluate the sources used, select and record information relevant to the enquiry and reach conclusions;

Geography: Collect, record and present evidence, analyse and evaluate evidence and draw and justify conclusions; **Citizenship:** Research a topical political, spiritual, moral, social or cultural issue, problem or event by analysing information from different sources, including ICT-based sources, showing an awareness of the use and abuse of statistics;

1. Specifying the problem and planning

In order to specify a problem, pupils need to suggest a conjecture (hypothesis) that could be investigated. A *conjecture* is a *hypothesis*. This means that it is a statement about something you're going to investigate, eg:

- tallest athletes jump best
- the cost of a car has an effect on its speed

2. Collecting Data

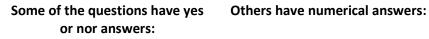
It is important that data are collected for a purpose. Data are found as either:

- a) Primary data data you collect yourself using a survey or experiment; or
- b) Secondary data data that is already collected for you. You can find secondary data in books or on the internet.

Example: Survey/Questionnaire

To decide whether traffic outside school can be reduced, the school governors want to ask drivers:

- How far is it to school?
- Do you drive in every day?
- Why do you drive your children to school?
- How long does your car journey take?
- How many people do you bring to school?
 - Do any other pupils live near you?
- What do you think of the traffic outside school?
- What buses go from near your house?



- Do you drive in every day?
- Do any other pupils live near you?



- How far is it to school?
- How long does your car journey take?
- How many people do you
 bring to school?

These have many different answers:

 Why do you drive your children to school?

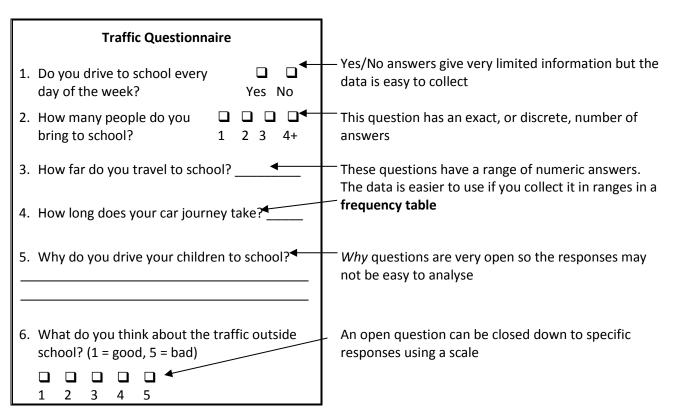
What buses go from near your house?

What is your opinion on the traffic outside school?

These are **closed** questions. They have particular answers. You could use tick boxes to collect this data

These are **open** questions. They can include answers you haven't thought of.

The governors develop a questionnaire for their questions:



You can use a questionnaire to conduct a survey. Open questions invite any response. Closed questions invite choice.

To understand how to collect data properly, it is necessary to consider different types of data, so that collection and handling activities can take place. One key idea, important to the proper collection of data, is that of sampling.

The Vocabulary of Sampling

Population: The entire group of people, animals, or things about which we want information

Sample: A part of the *population* from which we actually collect data/information, used to draw conclusions about the whole

IMPORTANT:

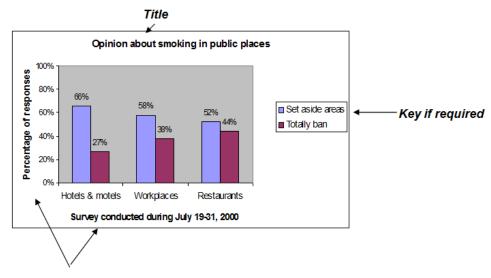
In order for a sample to be suitable, **at least 30**, pieces of information need to be collected.

3. Representing data and interpretation

Representing data in an orderly and easy-to-read/understand form is paramount to Handling Data. Charts and diagrams without headings, labels and an appropriate scale are useless. The representations synthesise the raw data into summary information. We will be looking at how to draw the most common charts: bar charts, pie charts and scatter diagrams. Also a brief look at averages.

Bar Charts:

A bar chart uses bars to represent data. Each bar represents a category or class. There must be GAPS between bars.



Both axes labelled

INTERPRETATIONS

- The bar chart shows that more people would set aside areas for smokers in public places than would ban them completely.
- > The more enclosed the space, the more would actually ban smoking totally
- None of the bars add to 100%, so it is assumed that the rest of the respondents 'didn't know' or perhaps they said smoking should be allowed everywhere.
- There is no information about who took part in the survey, such as whether it included smokers as well as nonsmokers, and so it is difficult to draw any firm conclusions.

Pie Charts:

A pie chart uses a circle to show data. Each class or category has a slice of the circle. **Example**: Draw a pie chart to illustrate the following information.

Type of transport	Train	Coach	Car	Ship	Plane
Frequency	48	28	125	22	27

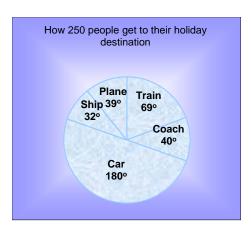
We need to find the fraction of the total, which represents each type of transport, to find the angle we then multiply by 360 degrees.

Type of transport	Frequency	Angle
Train	48	48 ÷ 250 x 360 ≈ 69°
Coach	28	$28 \div 250 \text{ x } 360 \approx 40^{\circ}$
Car	125	$125 \div 250 \text{ x } 360 \approx 180^{\circ}$
Ship	22	$22 \div 250 \times 360 \approx 32^{\circ}$
Plane	27	27 ÷ 250 x 360 ≈ 39°
Totals	250	360°

Notice:

- > We use the total of 250 to calculate each fraction
- > We round off each angle to the nearest degree
- \blacktriangleright We check that the sum of all the angles is 360°

The pie chart can now be drawn. Remember, it is always good practice to draw the smallest angle first, then the next smallest, and so on, until the last angle will automatically be the largest. This reduced the effect that the successive additions of error have on the accuracy of the last angle drawn.



INTERPRETATIONS

- Most people travel to holiday by car
- Less than a quarter go by train
- If 1000 people went on holiday, about 160 would go by coach

There is no information about who took part in the survey, so is the pie chart representative of the population?

Scatter diagrams:

A scatter diagram is a method of comparing two sets of data, and discovering if there is a link (relationship) between them.

Looking at relationships, scatter diagrams tell us whether there is a **correlation** (link) between the two data sets. It is quite common when using scatter diagrams to include a line of best fit (a straight line), which goes through the middle of the data, passing as close to as many points as possible. This would allow us to make estimates for certain cases.

Here are three statements that may or may not be true.

- > The taller people are, the wider their arm span is likely to be.
- > The older a car is, the lower its value will be.
- > The distance you live from your place of work will affect how much you can earn.

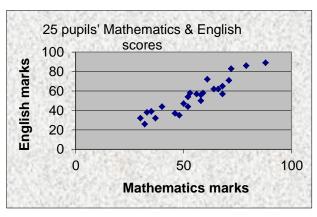
Collecting data and plotting the data on a scatter diagram could test these relationships. For example, the first statement may give a scatter diagram like that on the right. This has a **positive correlation** because the data has a clear 'trend' and we can draw a line of best fit that passes quite close to most of the points. From such a scatter diagram we could say that the taller someone is, the wider the arm span.

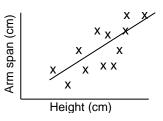
Testing the second statement may give a scatter diagram like that on the right. This has a **negative correlation** because the data has a clear 'trend', and we can draw a line of best fit that passes quite close to most of the points. From such a scatter diagram we could say that as a car gets older, its value decreases.

Testing the third statement may give a scatter diagram like that on the right. **This scatter diagram has no correlation.** It is not possible to draw a line of best fit. It could therefore say that there is no relationship between the distance a person lives from his or her work and how much the person earns.

IMPORTANT:

The line of best fit does not have to pass through (0, 0).









Distance from work

Averages:

This is a number that is used to represent a set of data. There are three main averages used in different circumstances. You have to choose the most appropriate average to use. When we talk about 'average' in everyday life we are using the mean.

MEAN: The sum of all the values divided by the number of values, eg Find the mean of 6, 3, 1, 4

Mean =
$$\frac{6+3+1+4}{4}$$

= 14 ÷ 4
= **3.5**

MEDIAN: The value in the middle of the data after it has been arranged in size order. If we have an even number of data, then we find the mean of the middle two values.

Example 1. Find the median of 4, 6, 3, 2, 1

Example 2. Find the median of 4, 6, 3, 2, 1, 2

6, 4,
$$(6, 2, 2, 1)$$
, 1 ∴ Median = $\frac{3+2}{2}$
= 2.5

Mode: The value in the data that occurs most frequently, e.g.

Find the mode of : 3, 15, 0, 3, 1, 0, 4, 3 Mode = 3

If there is no number that occurs most often, there is no mode.

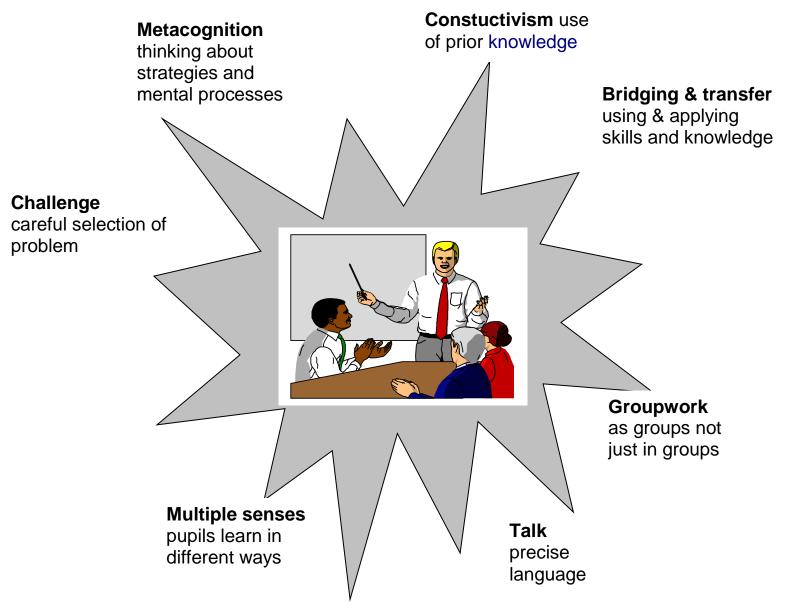
The range is the spread of data, i.e. the largest value subtract the smallest value, e.g.:

7, 6, 8, 12, 9 Range = 12 - 6 = 6

The mean is a good average when the range is small. The median is a useful average when the range is large.

PROBLEM SOLVING

Problem solving is a key area in mathematics, which is used often in other subject areas, just without recognition. This can be integrated into lessons by providing pupils with classroom activities that give pupils opportunities and time to think, discuss and reason. The **whole-school approach to problem solving** is intended to promote wider opportunities for pupils to develop their thinking skills by being given tasks that require *information processing, enquiry, critical thinking, reasoning and evaluation.*



ALGEBRA

Algebra is often referred to as the language of mathematics. When working with algebra it is important that before attempting to perform any calculations pupils translate the 'algebra' into 'English'.

E.g. 5x + 3 = 18 means "five times a number plus three equals eighteen"

Solving Equations

We have a consistent method in maths classes for solving equations, which is called the **balance method**.

Solve: 5x + 3 = 18

The idea here is to consider the equals sign as a set of balancing scales, and therefore whatever you do to one side of the equals sign you have to do to other if the scales are to remain balanced. For example, if you add 3 to one side you must add 3 to the other; if you divide by 4 on one side, you divide by 4 on the other.

So, to solve our equation:
$$5x + 3 = 18$$

(-3) (-3)
 $5x = 15$
(÷5) (÷5)
 $x = 3$

Pupils can check solutions by substituting answers back into the original equations. We have spent a lot of time stressing with pupils about the importance of setting out their work clearly, especially with equations.

Rearranging Formulae

1. Machine/Inverse Method

Rearrange the following equation, writing v in terms of u

$$v = \frac{u}{2} + 14$$

First we look at what is happening to u, in the correct order, to get v. We then reverse the flow diagram by putting the "v" back through the machine.

 $u \rightarrow \div 2 \rightarrow + 14 \rightarrow v$

Performing the inverse operations e.g. +14 becomes -14 and $\div2$ becomes $\times2$

$$2(v - 14) \leftarrow \times 2 \leftarrow (V - 14)$$

so $u = 2(v - 14)$

2. Balance Method

This follows the same steps as solving an equation using the balance method.

Example:

$$v = \frac{u}{2} + 14$$
(-14)
(-14)
$$v - 14 = \frac{u}{2}$$
(x2)
(x2)
$$2(v - 14) = u$$

Substituting into formulae

Again, it is essential that pupils write out what the formula means in "long hand", before replacing the letters with numbers. Stressing the importance of method is essential to obtaining the correct answer. It is expected that pupils show all of the following working out exactly as detailed below, the equal signs all underneath each other.

Example: v = u + at means ' $v = u + a \times t'$ (remembering to multiply first!)

So given u = 4, a = -5, t = 10, v = ?

We now *literally* replace the letters with the numbers and perform the calculation in the normal way, not forgetting to multiply first!

 $v = 4 + (-5) \times 10$ v = 4 + (-50)v = -46

ACCURACY IN MEASUREMENT AND DRAWING

Pupils should be expected to draw and measure accurately. It is an essential requirement in many subjects. For example:

- Reading scales in science and technology
- Measuring and cutting materials in textiles
- Plotting points on graphs in geography
- Measuring distances and times in physical education

Equipment:

Pupils should be actively encouraged to have with them at all times *a ruler, protractor, a pair of compasses, a sharp pencil and an eraser* so that they can work accurately.

Estimation:

Estimation is an important aspect of measurement and drawing. Pupils should be encouraged whenever possible to make sensible estimates before measurement. Estimation can help pupils avoid careless mistakes in measurement. Estimation can also be used to introduce discussion on appropriate and sensible degrees of accuracy.

Units:

The choice of units is also important particularly as many pupils confuse the units of length, area and volume. Pupils also need to understand that in some contexts, millimetres are used as the principle unit of length rather than centimetres. Note: pupils are taught about commonly used imperial units and their metric conversions.

Checking accuracy:

Pupils involved in measurement tasks need to be clear about the level of accuracy required so their work can be checked and marked fairly. Peer assessment is a very useful strategy for improving accuracy and promotes self-evaluation.

Tables, Charts and graphs:

For consistency and accuracy, drawing will usually be done in pencil, with straight lines drawn using a ruler, for *example, in tables, graph axes, sketches and diagrams.* Points and lines on graphs should be plotted and drawn using a sharpened pencil. Labelling of graphs and diagrams should normally be completed in ink, for example titles, axes labels etc. The use of a pencil and eraser can be a helpful way to improve drawing, for example in drawing curved line graphs. Sketches that do not need to be accurately measured, still need to be neat and legible (as does numerical working and jottings).

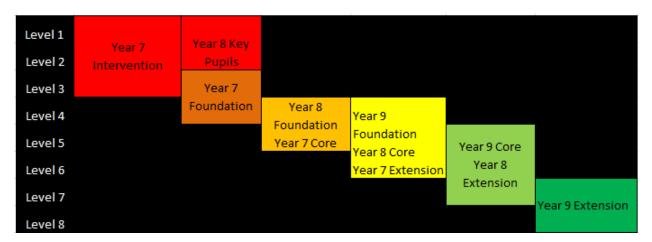
APPENDIX ONE

Below you will find the general band of levels your pupils are working at in maths and key mathematical skills that pupils generally develop throughout Key Stages 3. This will also be a guide for teachers to see when certain key topics are taught.

Please note:

Pupils are expected to explain their methods and reasoning at every opportunity.

Levels each class is approximately working at:



Key skills of year 7 foundation and year 8 very weak students

	Number	Calculation	Algebra	Shape and space	Handling data
Level 1	Counting to 10	+ and - to 10			
Level 2	Counting to 100	+ and - mentally	odd and even numbers	name 2D and 3D shapes	Sorting objects, record results, draw pictograms
Level 3	Counting to 1000, decimals in money		role of '=' sign	use time, metric units of length, mass, capacity	bar charts, venn diagrams
Level 4	x and / by 10/100	x and / of numbers by single digit numbers	plotting positive coordinates	make nets, find perimeter, reflect shapes,	group data, mode and range

Key skills of year 7 core and year 8 foundation

	Number	Calculation	Algebra	Shape and space	Handling data
Level 4	x and / by 10/100	x and / of numbers by single digit numbers	plotting positive coordinates	make nets, find perimeter, reflect shapes,	group data, mode and range
Level 5	simple ratio	Estimating, solving problems with negative numbers,	plotting negative points, formulae with one operation or word formulae	measuring and drawing angles, read and interpret scales on real- life objects, area of rectangle	write survey, median and mean, pie charts, line graphs

Key skills of year 7 extension, year 8 core and year 9 foundation

	Number	Calculation	Algebra	Shape and space	Handling data
Level 4	x and / by 10/100	x and / of numbers by single digit numbers	plotting positive coordinates	make nets, find perimeter, reflect shapes,	group data, mode and range
Level 5	simple ratio	Estimating, solving problems with negative numbers,	plotting negative points, formulae with one operation or word formulae	measuring and drawing angles, read and interpret scales on real- life objects, area of rectangle	write survey, median and mean, pie charts, line graphs
Level 6	compare fractions, decimals and percentages	Calculating with percentages, fractions and ratio	solving equations, plotting straight line graphs	area and circumference of a circle, area of triangle, volume of cuboid	scatter graphs,

Key skills of year 8 extension and year 9 core

	Number	Calculation	Algebra	Shape and space	Handling data
Level 5	simple ratio	Estimating, solving problems with negative numbers,	plotting negative points, formulae with one operation or word formulae	measuring and drawing angles, read and interpret scales on real- life objects, area of rectangle	write survey, median and mean, pie charts, line graphs
Level 6	compare fractions, decimals and percentages	Calculating with percentages, fractions and ratio	solving equations, plotting straight line graphs	area and circumference of a circle, area of triangle, volume of cuboid	scatter graphs,
Level 7		Use calculator for complex calculations	subsitute into formulae from different subjects, plotting curves	enlarge a 2D shape, use speed, density and pressure to solve problems	averages from grouped data

Key skills of year 9 extension

	Number	Calculation	Algebra	Shape and space	Handling data
Level 7		Use calculator for complex calculations	subsitute into formulae from different subjects, plotting curves	speed_density_and	averages from grouped data
Level 8		Calculate with powers, standard form	changing subject of formulae, sketching graphs that model real life situations	trigonometry	